

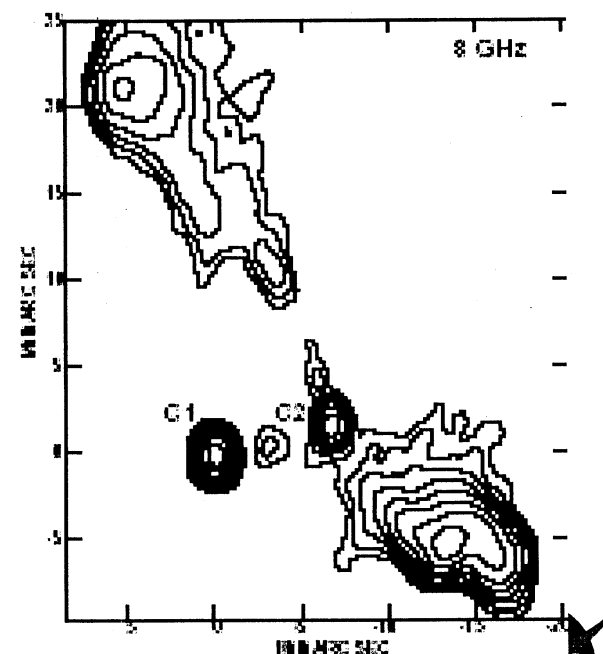
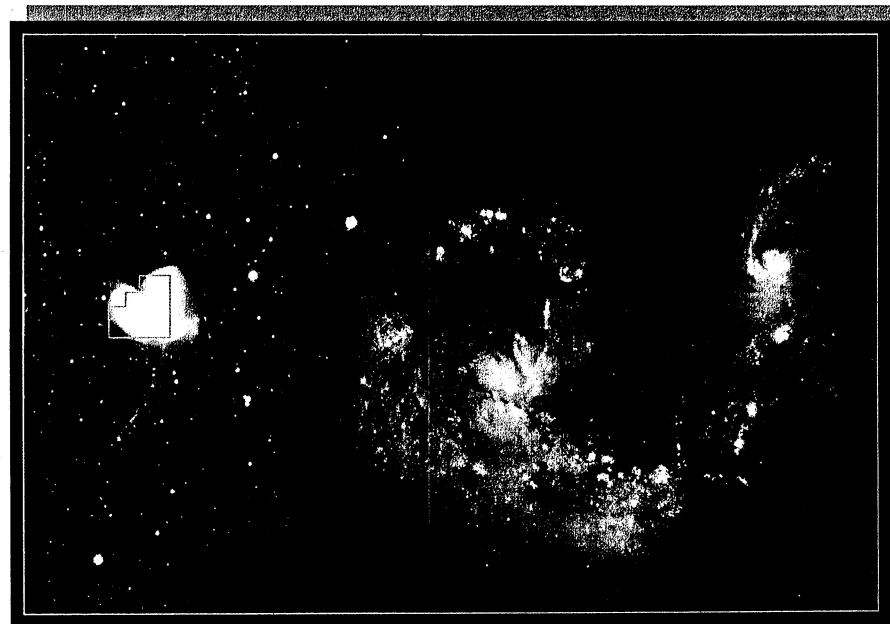
# ***Binary Black Holes, Gravitational Waves, and Numerical Relativity***

**Joan Centrella**  
**NASA/GSFC**

**Institute for Theory and Computation Colloquium  
Harvard-Smithsonian Center for Astrophysics  
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# ***MBH binaries....***

- MBHs are found at the centers of most galaxies
- Most galaxies merge one or more times  
→ *MBH binaries*
- MBH mergers trace galaxy mergers
- MBH mergers are strong sources of gravitational waves
- These GWs are detectable by LISA out to  $z \sim 10$  or more
- Expect  $\sim$  several events/year, or more (possibly more...)
- Observing these GWs can probe early stages of structure formation

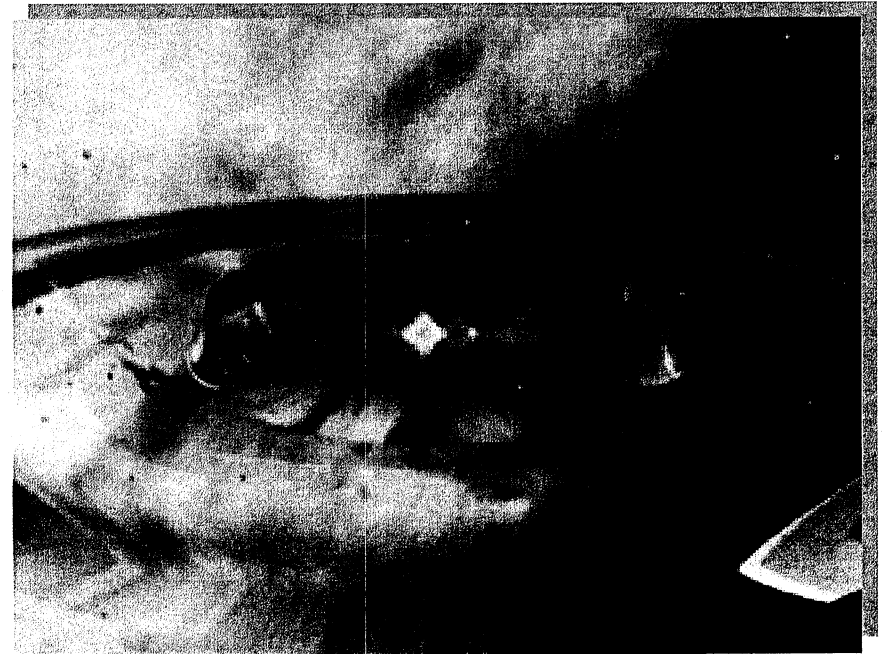


(Rodriguez, et al. ApJ,  
astro-ph/0604042, VLBA)

# A Different Type of Astronomical Messenger

## *Gravitational Waves . . .*

- Ripples in spacetime curvature
- Travel at velocity  $v = c$
- Generated by masses with time changing quadrupolar moments
- Carry energy and momentum
- Interact weakly with matter
- *carry info about deep, hidden regions in the universe*
- First *indirect* detection of GWs:  
Hulse-Taylor binary pulsar  
PSR 1913+16
  - Orbital period decay agrees with GR to within the observational errors of  $< 1\%$
  - Nobel Prize 1993

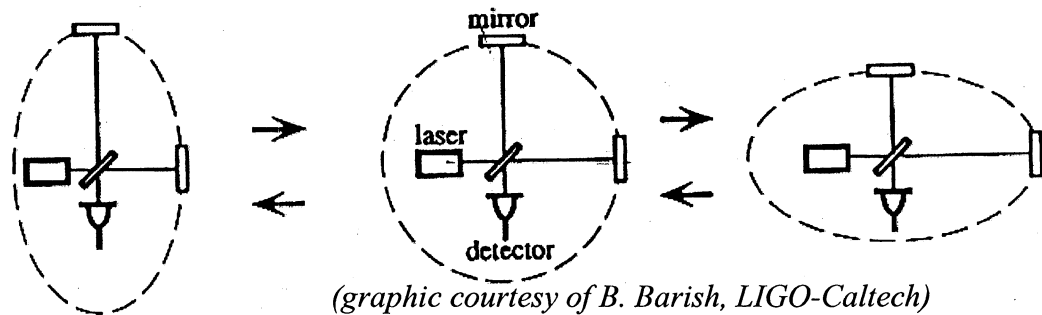
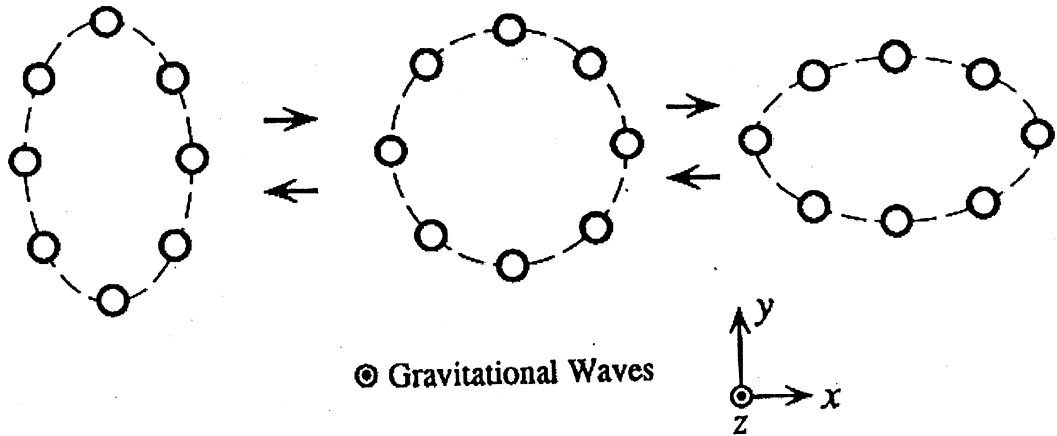


- The *direct* detection of gravitational waves will open a fundamental new window on the universe...



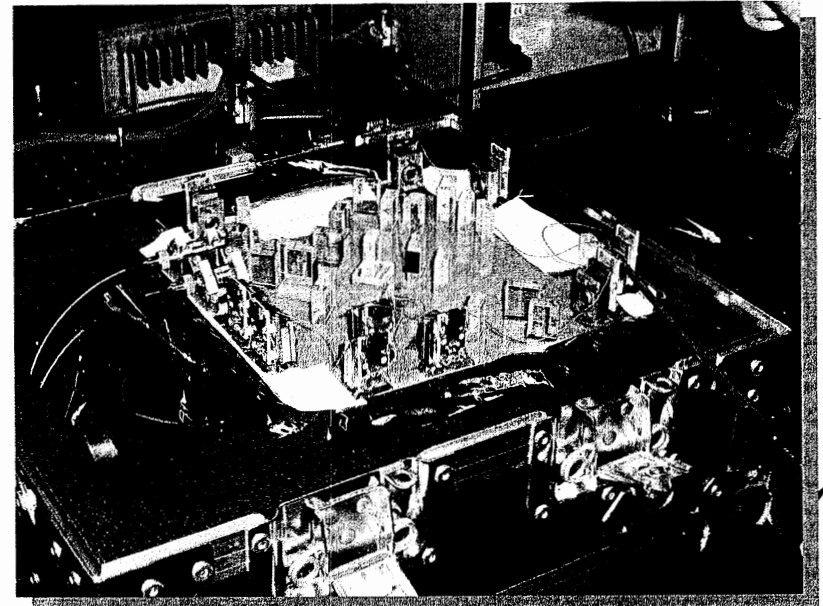
# Detecting gravitational waves. . .

- Detector of length scale  $L$
- A passing gravitational wave distorts detector via 2 polarization states,  $h_+$  and  $h_x$
- Measure strain amplitude  $h(t) = \Delta L/L$
- Source waveforms scale as  $h(t) \sim 1/r$



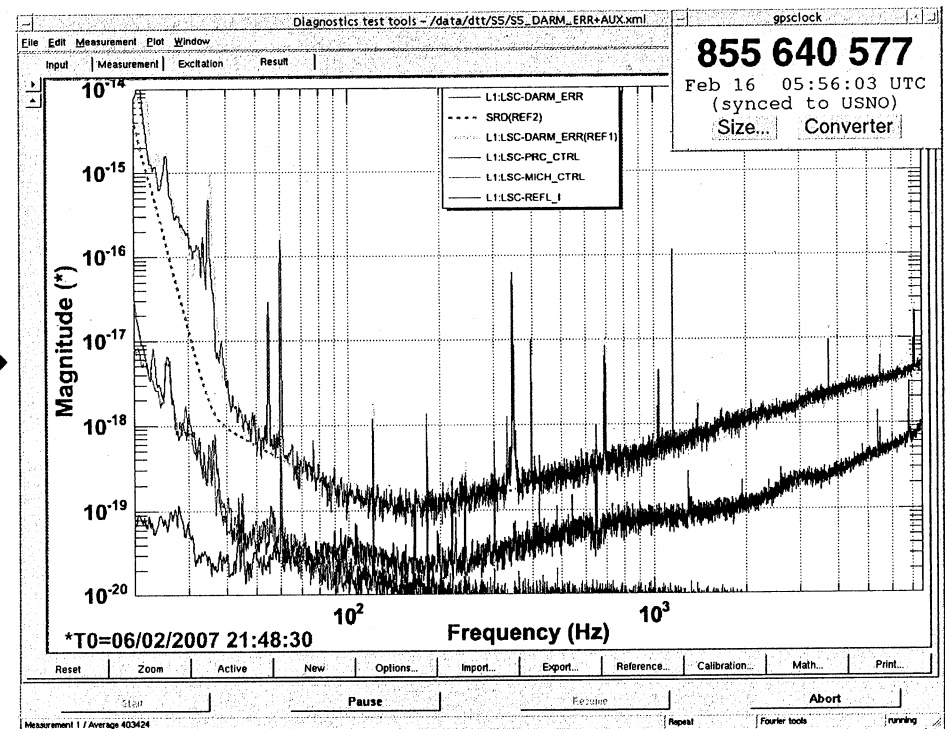
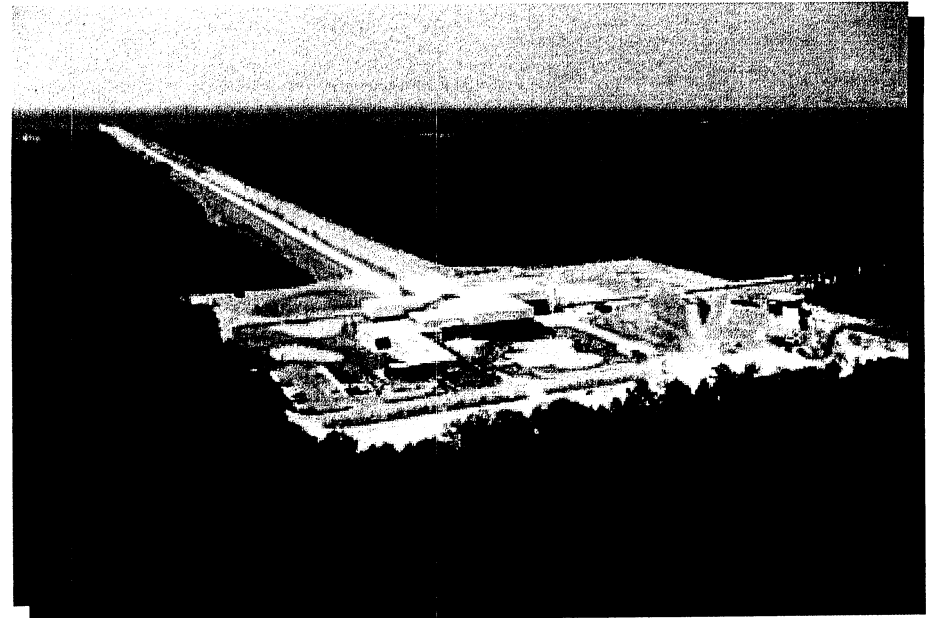
# *LISA: Laser Interferometric Space Antenna*

- NASA/ESA collaboration
- GSFC & JPL partnership
- detect *low frequency* GW  
 $10^{-4} \text{ Hz} \leq f_{\text{GW}} \leq 1 \text{ Hz}$
- typical sources: MBH/MBH binaries, galactic compact binaries, extreme mass ratio binaries...
- 3 spacecraft in equilateral triangle
  - orbits Sun at 1 AU
  - $20^\circ$  behind Earth in its orbit
- arm length  $L = 5 \times 10^6 \text{ km}$
- optical transponders receive & re-transmit phase locked light
- precision measurements:  
strain amplitude  $h = \Delta L/L < 10^{-20}$



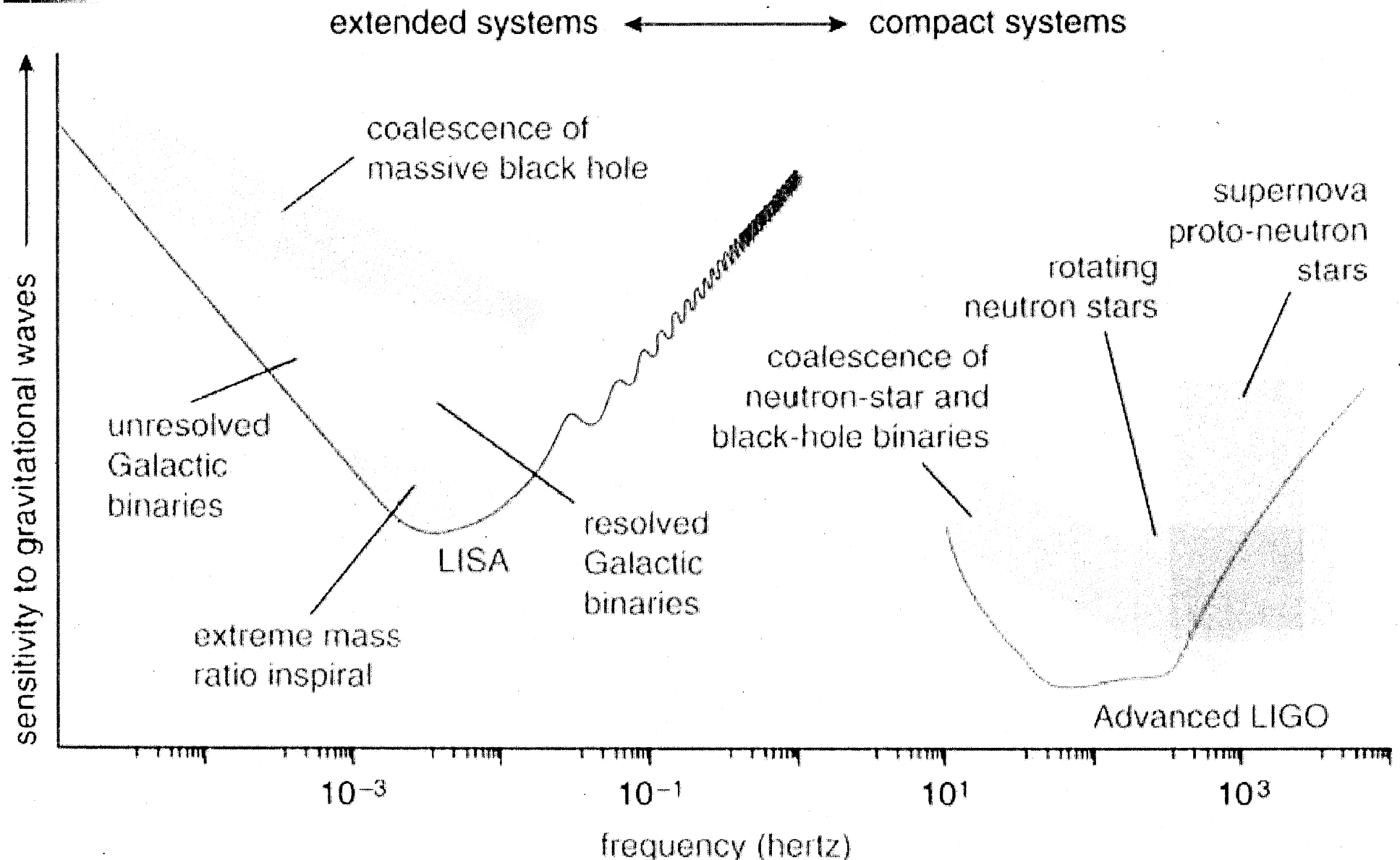
## Ground-based detectors . . .

- detect high frequency GW  
 $10 \text{ Hz} \leq f_{\text{GW}} \leq 10^4 \text{ Hz}$
- kilometer-scale arms
  - LIGO: Hanford, WA, and Livingston, LA;  $L = 4 \text{ km}$
  - VIRGO: PISA,  $L = 3 \text{ km}$
  - GEO600: Hannover  $L = 600 \text{ m}$
- Typical sources: NS/NS, NS/BH, BH/BH, stellar collapse...
- LIGO/GEO currently in year-long science data-taking run....
- Current sensitivity of detector →
- Available in read-only mode daily at <http://ilog.ligo-la.caltech.edu/>  
 (see the Detector Group Log, Figure of Merit 4)

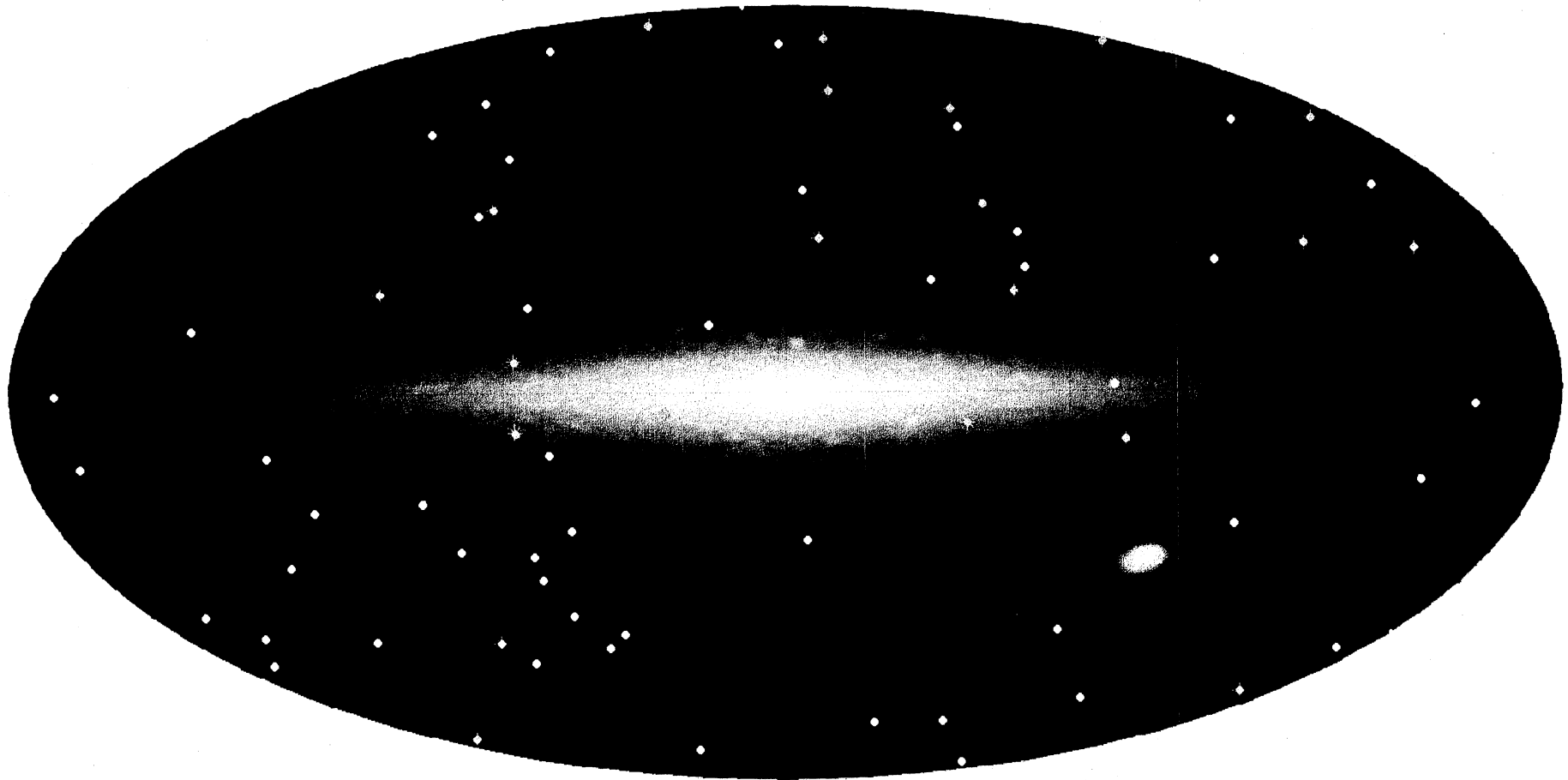


# Gravitational Wave Spectrum...

• Complementary observations, different frequencies & sources...



# *Simulation of the GW sky in the LISA band....*



<http://www.lisa-science.org/resources/talks-articles/science>

*Ground-based detectors will also see NS and stellar  
BH binary coalescences, supernovae...*





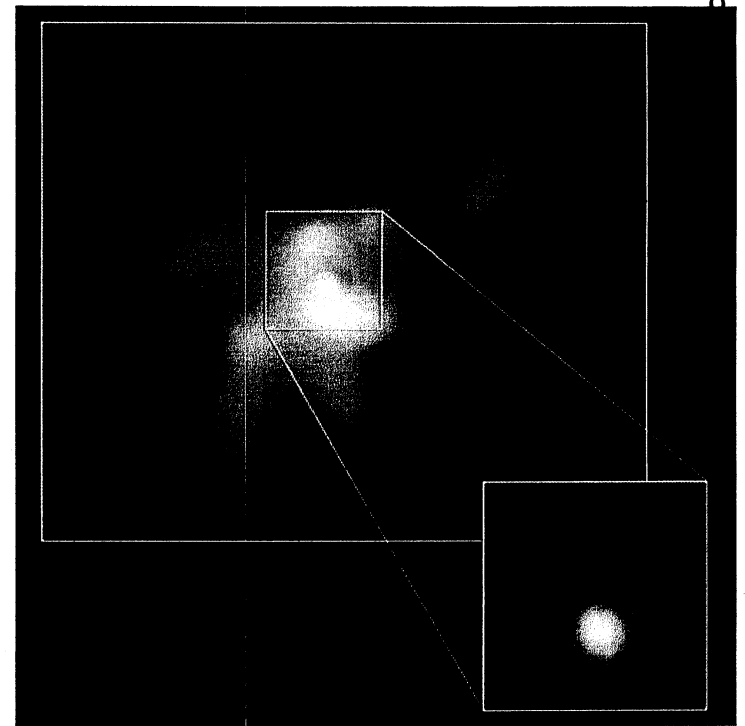
## *MBH mergers...*

- Final merger of MBHs occurs in the arena of very strong gravity
- Gravitational waves encode the dynamics of massive objects
- Observing GWs allows *direct* confrontation of GR w/ observations
- MBH mergers are strong GW sources
- *LISA can confront GR with*

*observations in the dynamical, strong field regime...*

*.... if we know the merger waveforms*

- When MBHs are spinning, and/or  $m_1 \neq m_2$ , the GW emission is asymmetric  $\rightarrow$  recoil kick
- If this kick is large enough, it could eject the merged remnant from the host structure... and *affect the rates of merger events*
- MBH mergers could produce *interesting spin dynamics*

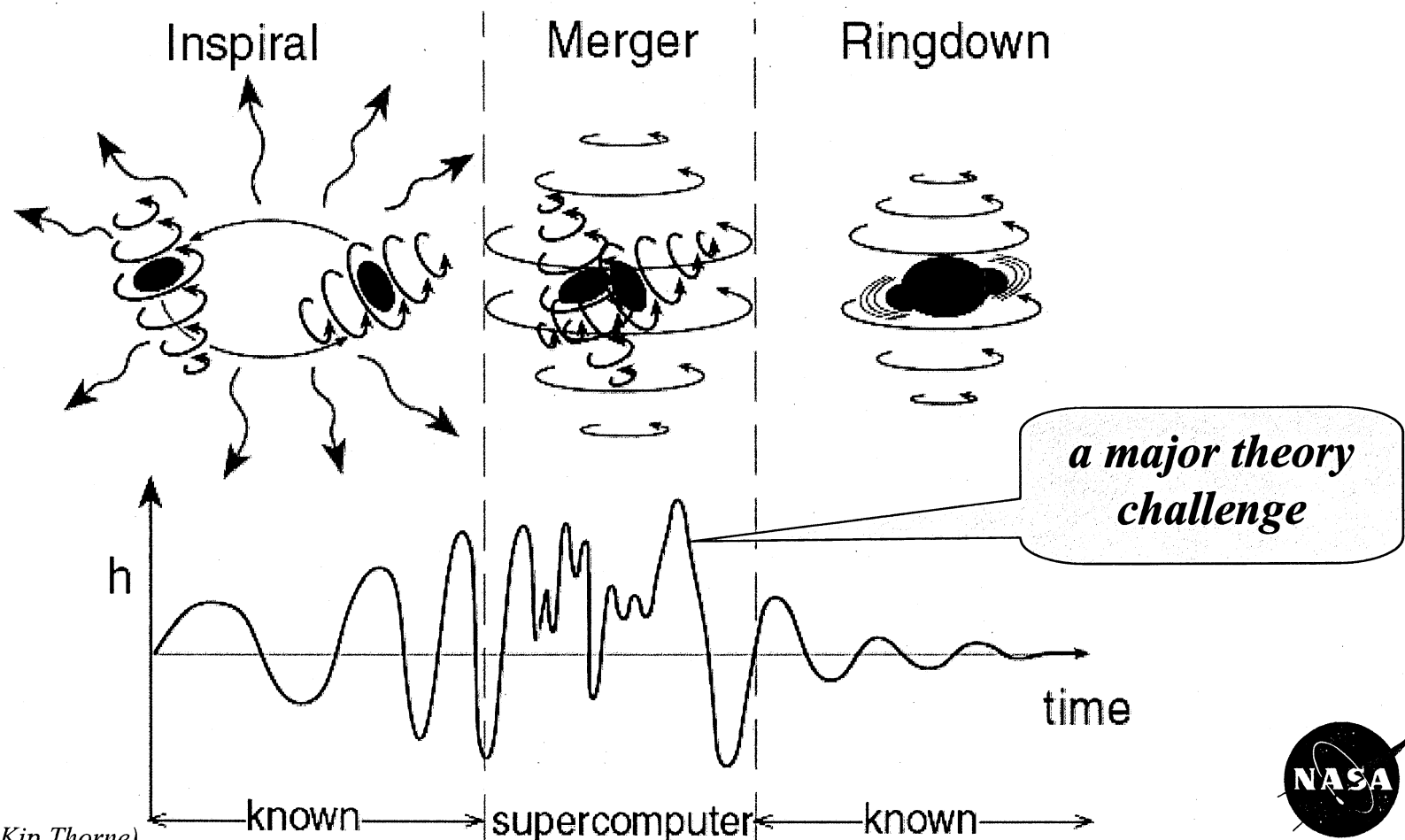


(NASA/CXC/MPE/S.Komossa et al. )



# *GWs from final merger of black hole binary...*

- Strong-field merger is brightest GW source, luminosity  $\sim 10^{23} L_{\text{SUN}}$
- Requires *numerical relativity* to calculate dynamics & waveforms
- Waveforms scale w/ masses, spins  $\rightarrow$  apply to ground-based & LISA



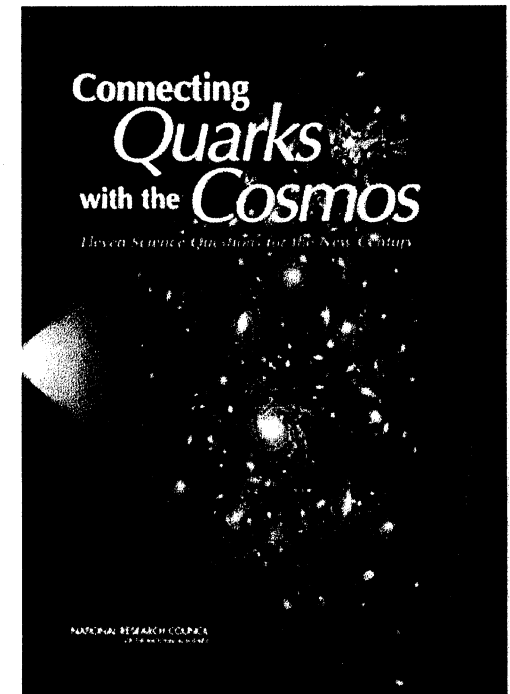
(graphic courtesy of Kip Thorne)



## *A major theory challenge....*

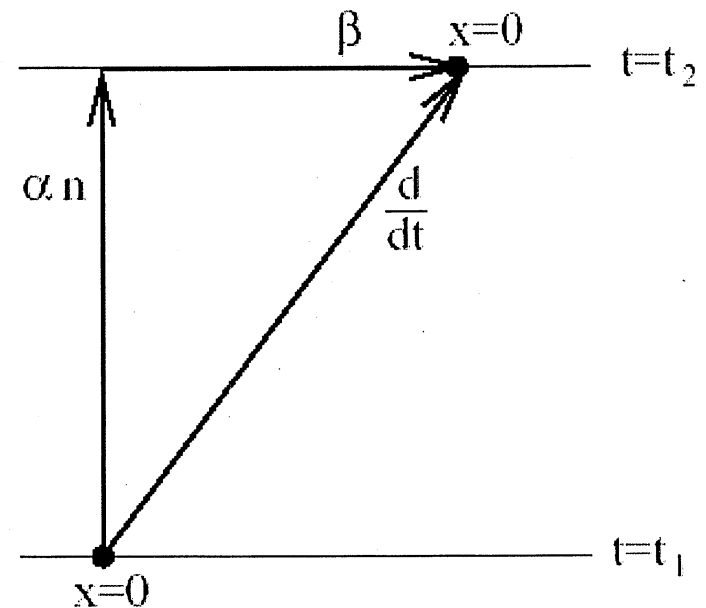
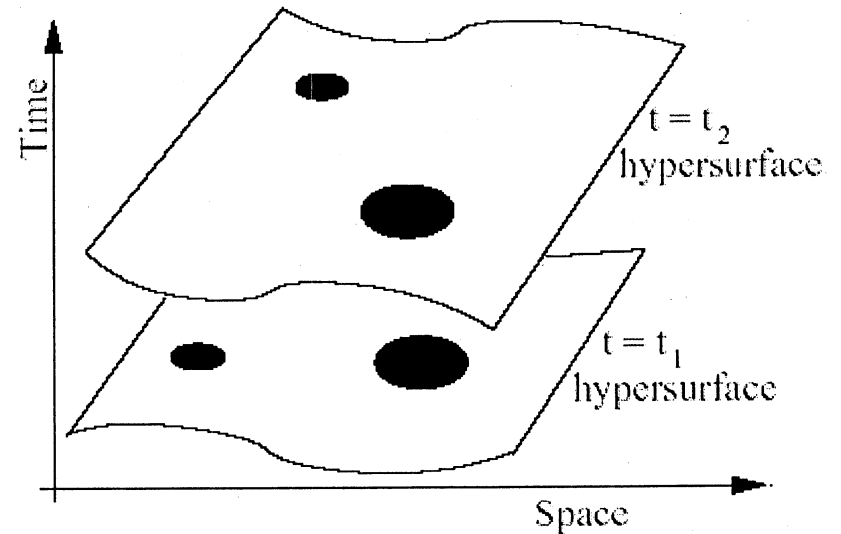
“Nearly as difficult as building these (*gravitational wave*) observatories, however, is the task of **computing the gravitational waveforms** that are expected when two black holes merge. This is a **major challenge** in computational general relativity and one that will stretch computational hardware and software to the limits. However, a bonus is that the **waveforms will be quite unique to general relativity**, and if they are reproduced observationally, scientists will have performed a **highly sensitive test of gravity in the strong-field regime.**”

-- “What are the Limits of Physical Law?”  
in *Connecting Quarks with the Cosmos:  
Eleven Science Questions for the New Century*  
(Board on Physics and Astronomy,  
National Academies, 2003), p. 118.



# *Numerical Relativity....*

- Solve Einstein eqns numerically
- Spacetime sliced into 3-D  $t = \text{constant}$  hypersurfaces
- Einstein's eqns split into 2 sets:
  - Constraint equations
  - Evolution equations
- Constrained initial data at  $t = 0$
- Evolve forward in time, from one slice to the next
- Typically solve 17 or more nonlinear, coupled PDEs
- Coordinate or gauge conditions: relate coordinates on neighboring slices
  - lapse function  $\alpha$ , shift vector  $\beta^i$



## *A Brief History of BBH simulations....*

- 1964: Hahn & Lindquist: try to evolve collision of 2 “wormholes”
- 1970s: Smarr and Eppley: head-on collision of 2 BHs, extract GWs
  - Pioneering efforts on supercomputers at Livermore Natl Lab
- 1990s: LIGO moves ahead & work on BBH problem starts up again..
  - Work on 2-D head-on collisions at NCSA
  - NSF Grand Challenge: multi-institution, multi-year effort in 3-D
- *This is really difficult! Instabilities, issues in formalisms, etc...*
  - Diaspora: multiple efforts (AEI, UT-Austin, PSU, Cornell...)
  - Difficulties proliferate, instabilities arise, codes crash....
  - “*Numerical relativity is impossible...*”
- 2000s: LIGO/GEO/VIRGO and LISA spur more development
  - New groups arise: Caltech, UT-Brownsville, LSU, NASA/GSFC...
- Since 2004.....
  - Breakthroughs & rapid progress throughout community
  - Orbits, waveforms, and astrophysical applications....



# *Issues and ingredients for success...*

## Formulations of the Einstein equations

- fully 2<sup>nd</sup> order, fully 1<sup>st</sup> order, mixed 1<sup>st</sup> and 2<sup>nd</sup> order PDEs
- which variables to use?
- incorporate constraints into evoln eqns? solve constraints?

## Coordinate conditions:

- lapse function  $\alpha$  – “singularity avoiding” time slicing
- shift vector  $\beta^i$  – keep coordinates from falling into black holes...

## Constrained initial data to approximate astrophysical binary

- start on approx quasi-circular orbits
- inward radial velocity...

## How to handle the black holes:

- excision? punctures?
- comoving coordinates? move the black holes?

## Variable grid resolution to handle multiple scales:

- $\lambda_{\text{GW}} \sim (10 - 100)M$
- $c = G = 1 \rightarrow 1 M \sim 5 \times 10^{-6} (M/M_{\text{Sun}}) \text{ sec} \sim 1.5 (M/M_{\text{Sun}}) \text{ km}$
- finite differences w/ mesh refinement; spectral methods



# *The 1<sup>st</sup> complete BBH orbit...*

Bruegmann, Tichy, & Jansen, PRL,  
92, 211101 (2004), gr-qc/0312112

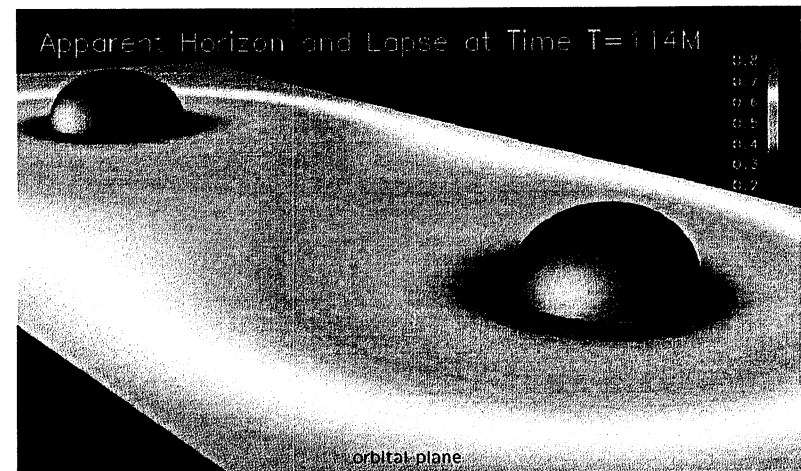
- equal mass, nonspinning BHs
- Represent BHs as “punctures”:

$$g_{ij} = \psi^4 \delta_{ij} \quad \psi = \psi_{BL} + u$$

$$\psi_{BL} = 1 + \sum_{n=1}^2 m_n / 2 |r - r_n|$$

- Handle singular  $\psi_{BL}$  analytically;  
evolve only nonsingular  $u$   
→ fix the BH punctures in the grid
- Use comoving shift vector  $\beta$
- Conformal formalism
  - $g_{ij}, A_{ij} \sim \partial_t g_{ij}$
  - 1<sup>st</sup> order time, 2<sup>nd</sup> order space

- Traditional numerical relativity techniques
- Excise BHS at late times

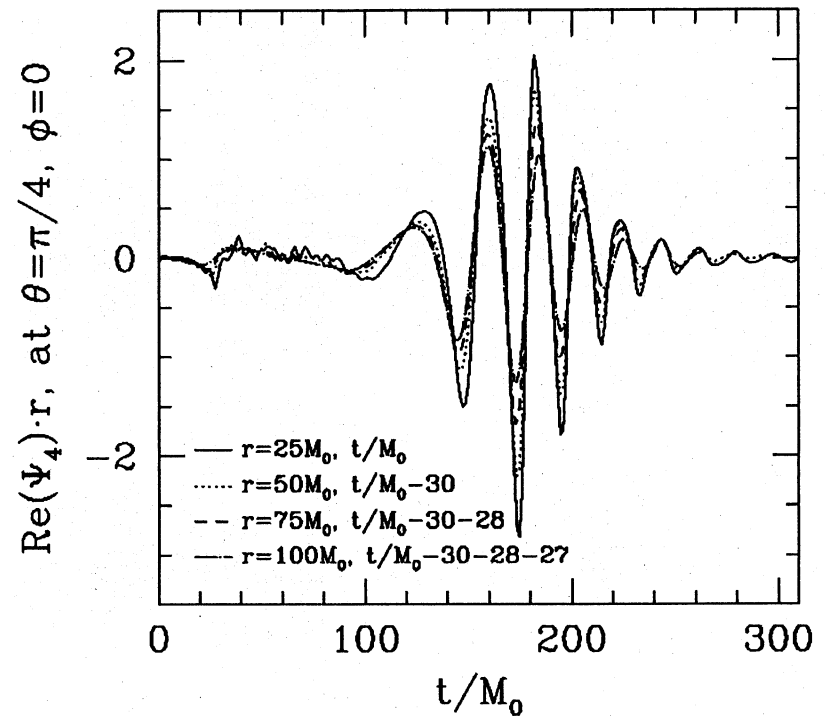


- Ran for  $\sim (125 - 150)M$  and BHs completed  $\sim 1$  orbit
- Crashed before BHs merge
- Not accurate enough to be able to extract GWs



# *The 1<sup>st</sup> orbit, merger, & ringdown...*

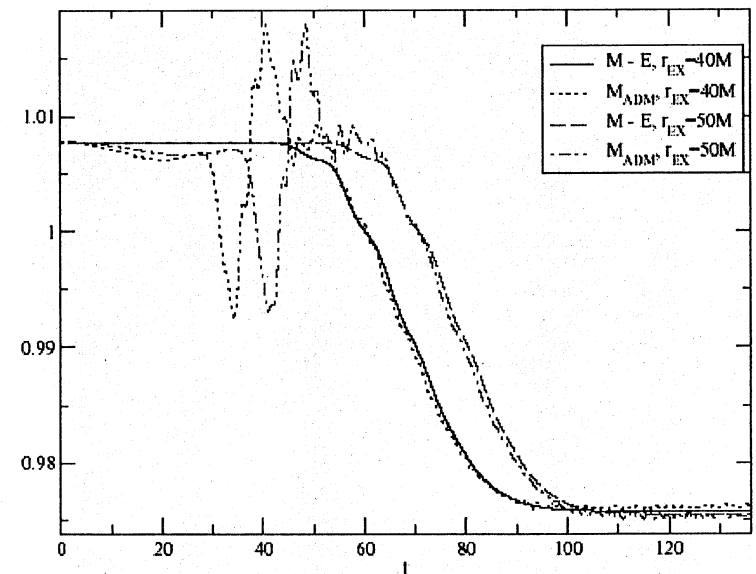
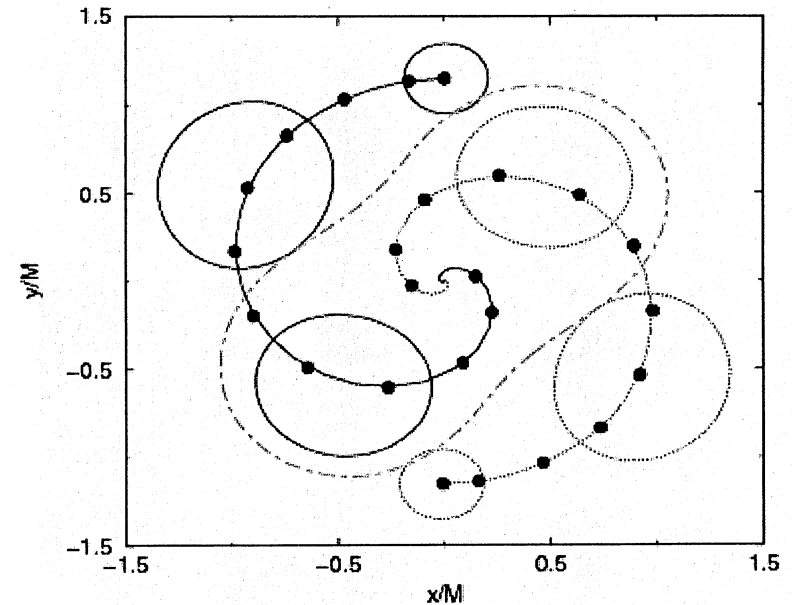
- Pretorius, PRL, 95, 121101 (2005)  
gr-qc/0507014
- Different formalism: based on “generalized harmonic coords”
  - metric  $g_{ij}$  is basic variable
  - 2<sup>nd</sup> order in space & time
- Excised BHs move through grid
- AMR: high resolution around BHs, tracks BHs as they move
- “Compactified” outer boundary: edge of grid at spatial infinity
- Equal mass, nonspinning BHs
- Start with 2 “blobs” of scalar field that collapse to BHs, then complete  $\sim 1$  orbit
- Indiv BH mass  $M_0$  ( $M \sim 2M_0$ )
- Show waveforms extracted at different radii (scaled)
- $\text{Re}(\Psi_4) \sim d^2/dt^2 (h_+)$





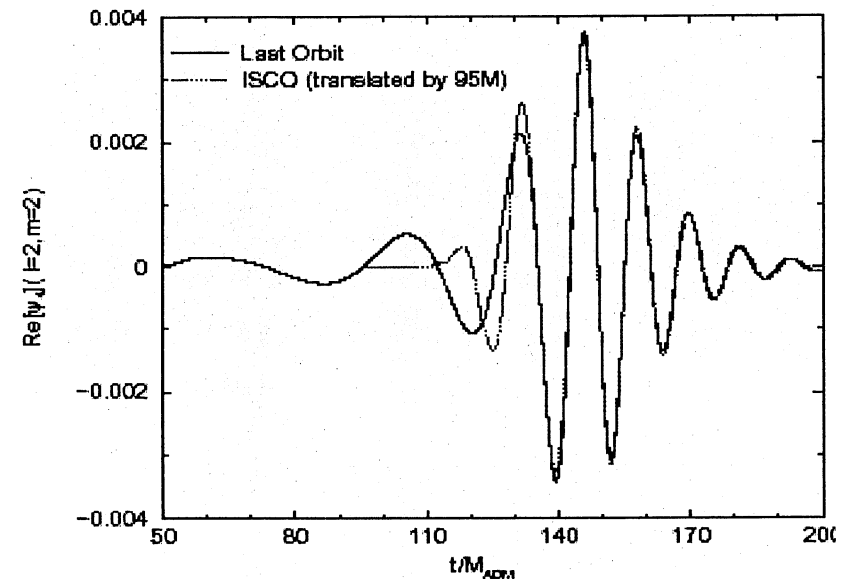
## *A new idea: “moving puncture BHs”*

- Allow puncture BHs to move across grid w/out excision
- Simultaneous, independent discovery by UTB & GSFC groups:
  - Campanelli, et al., PRL, 96, 111101 (2006), gr-qc/0511048
  - Baker, et al., PRL, 96, 111102 (2006), gr-qc/0511103
- Do not split off singular part  $\Psi_{BL}$ 
  - Regularize near puncture
  - New conditions for  $\alpha$  &  $\beta^i$
- Uses conformal formalism
- Enables long duration, accurate simulations



# *A powerful new idea....that spread rapidly*

- Developed w/in “traditional” numerical relativity approach:
  - Conformal formalism, BHs represented as punctures
- A simple, powerful new idea: allow the punctures to move
- Requires novel coordinate conditions:
  - Van Meter, et al., “How to move a puncture black hole without excision...,” PRD 73 (2006) 124011 (2006), gr-qc/0605030
- UTB, GSFC moved ahead rapidly, quickly able to do multiple orbits
- Moving punctures quickly adopted by other groups:
  - PSU, AEI/LSU, FAU/Jena...
  - At April 2006 APS meeting, a full session devoted to BBH mergers w/ moving punctures!
  - Summer 2006: method adopted by most of community



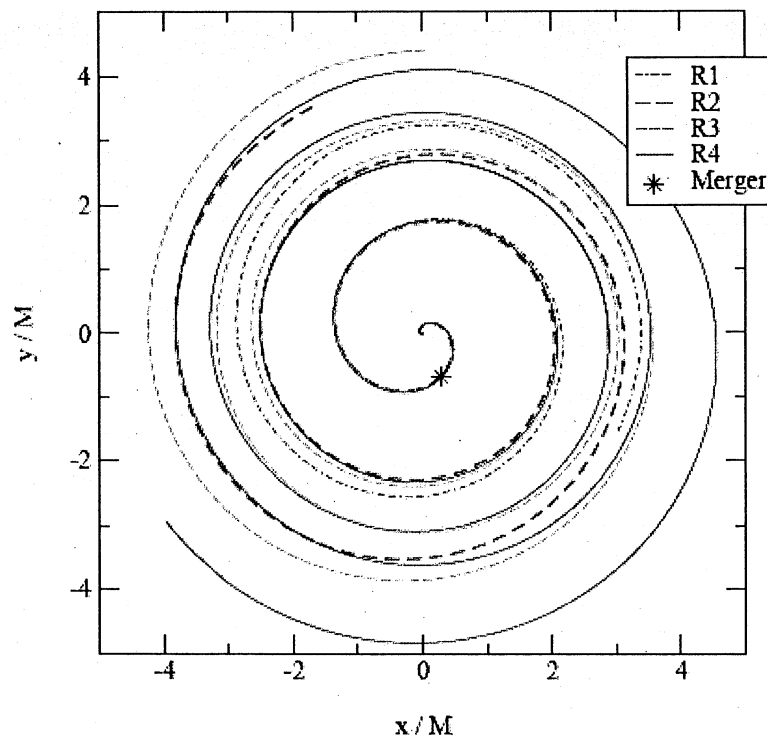
Campanelli, et al., PRD, 73, 061501 (2006), gr-qc/06010901



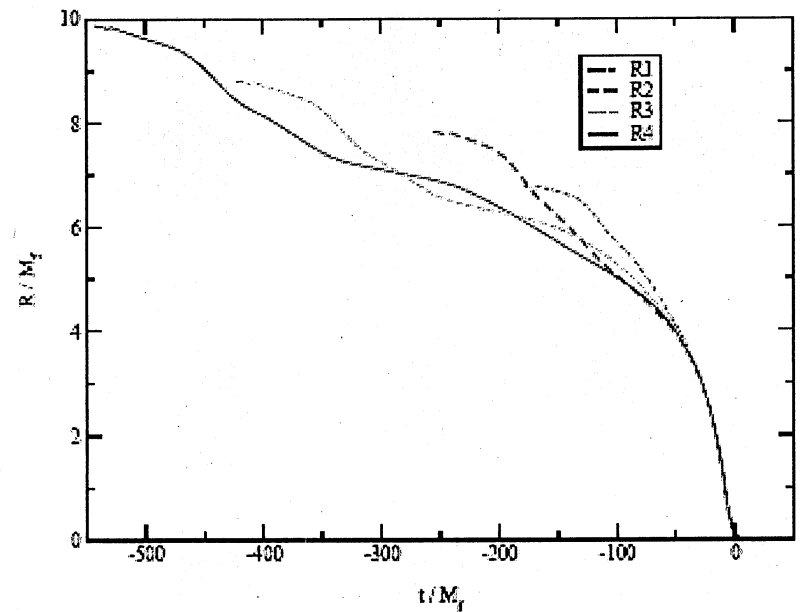
# *Revealing universal behavior...*

- Baker, al., PRD, 73, 104002 (2006), gr-qc/0602026
- Long duration simulations of moving punctures with AMR
- Equal mass, nonspinning BHs
- Run several cases, starting from successively wider separations
- BH orbits lock on to universal trajectory  $\sim$  one orbit before merger

BH trajectories (only 1 BH shown)

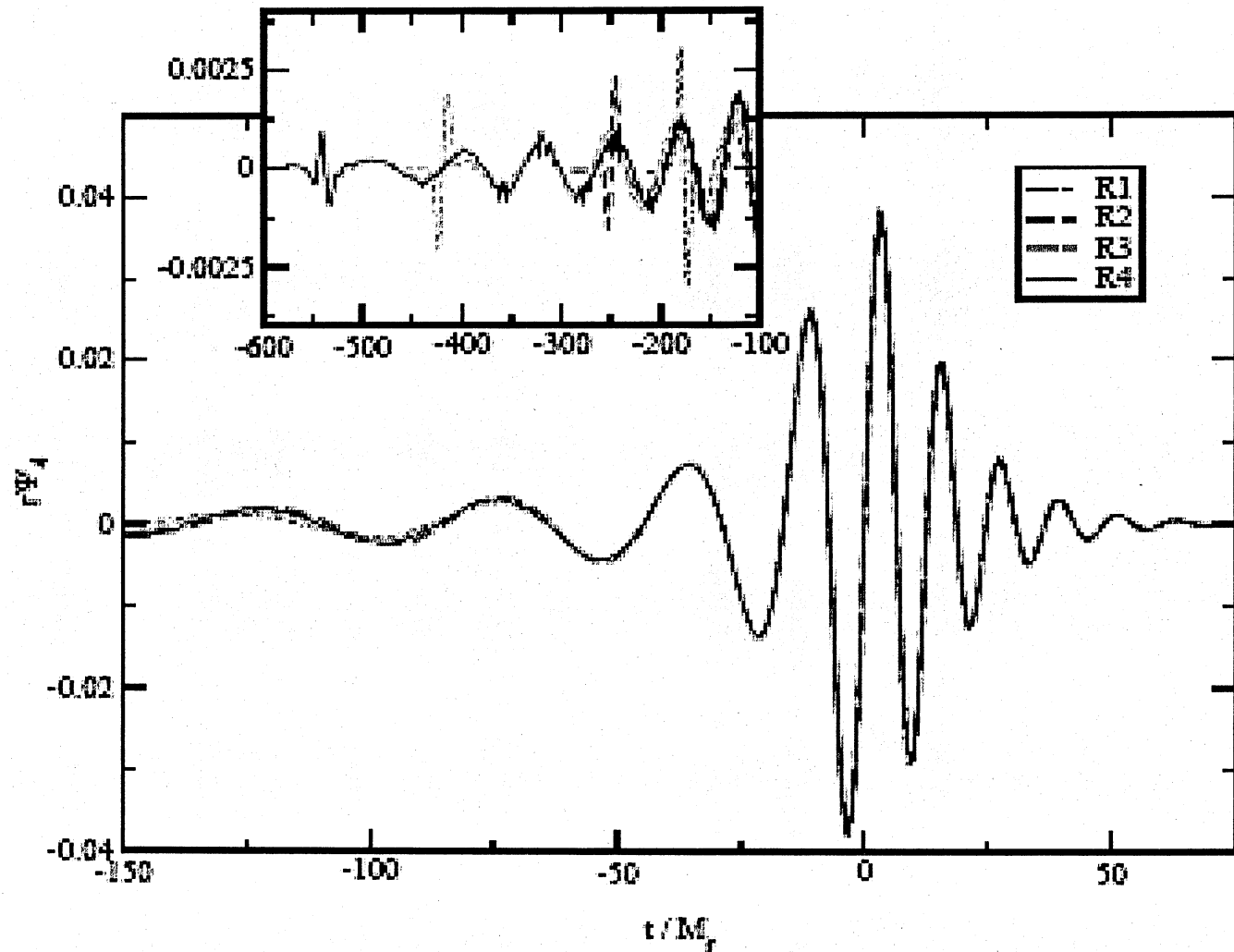


BH separation vs. time

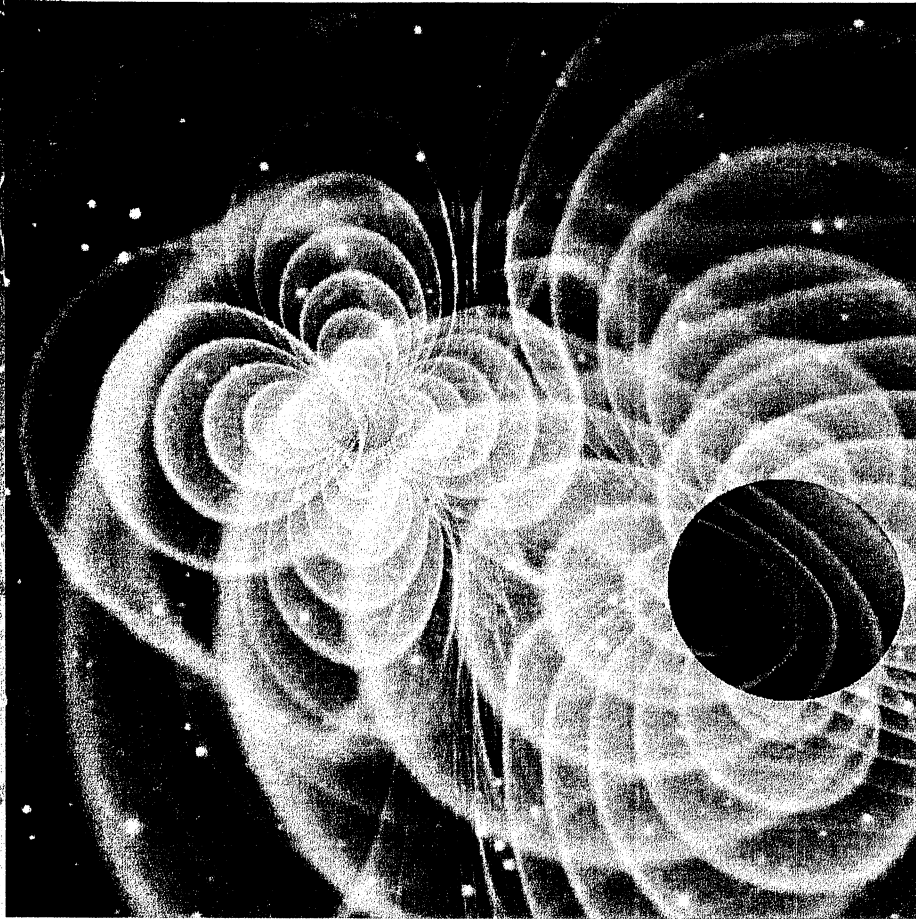


## *Universal waveform....*

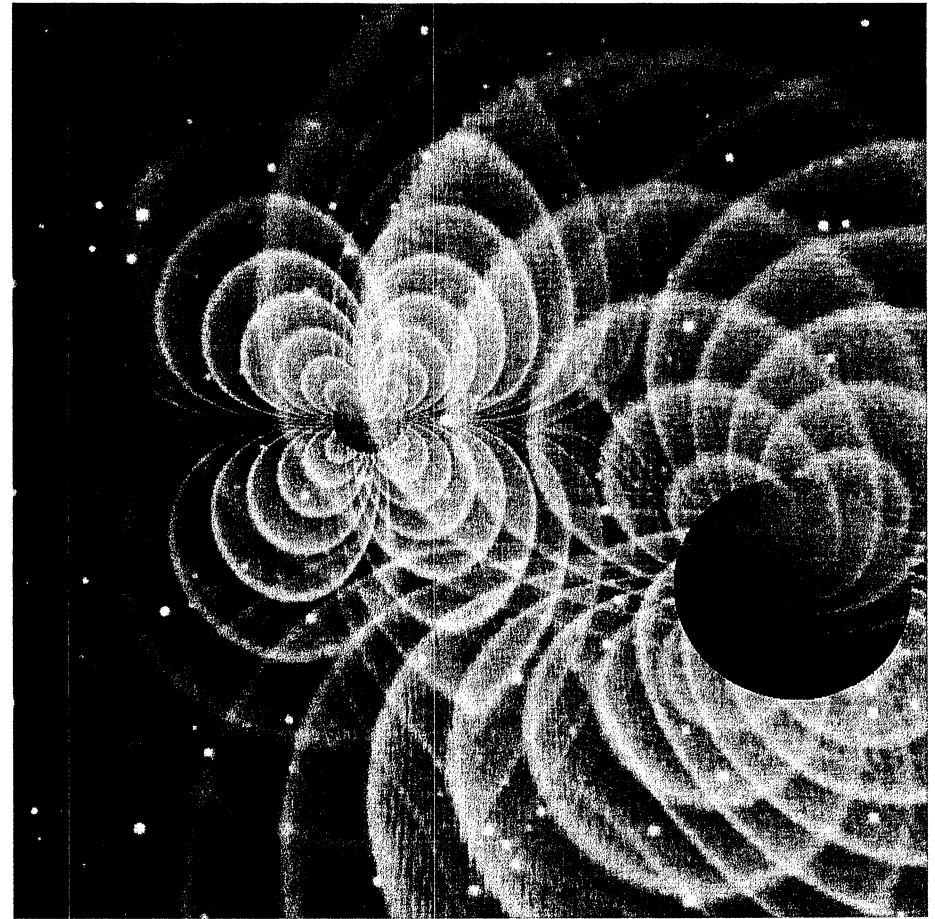
- Universal dynamics produces universal waveform....
- All runs agree to within  $< 1\%$  for final orbit, merger & ringdown



# *Binary Black Holes: The Movies*



$$\text{Re}[\psi_4] \sim d^2/dt^2 h_+$$



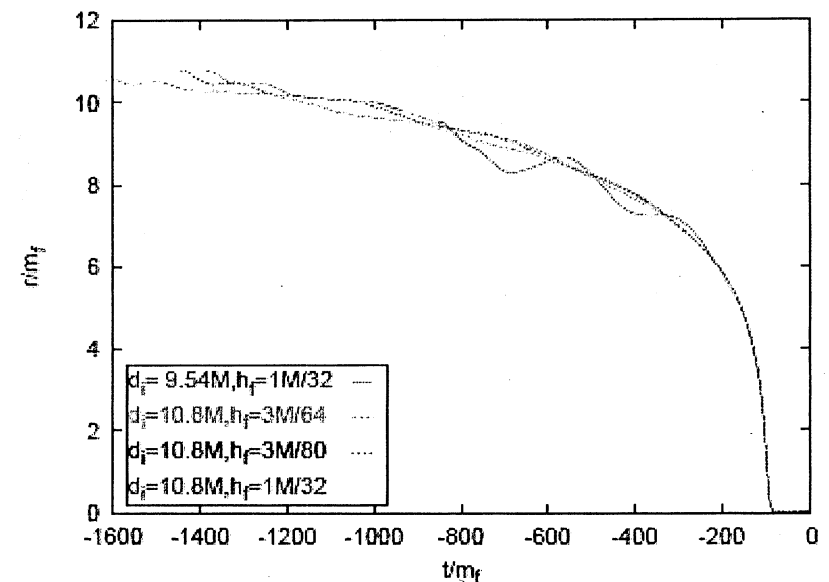
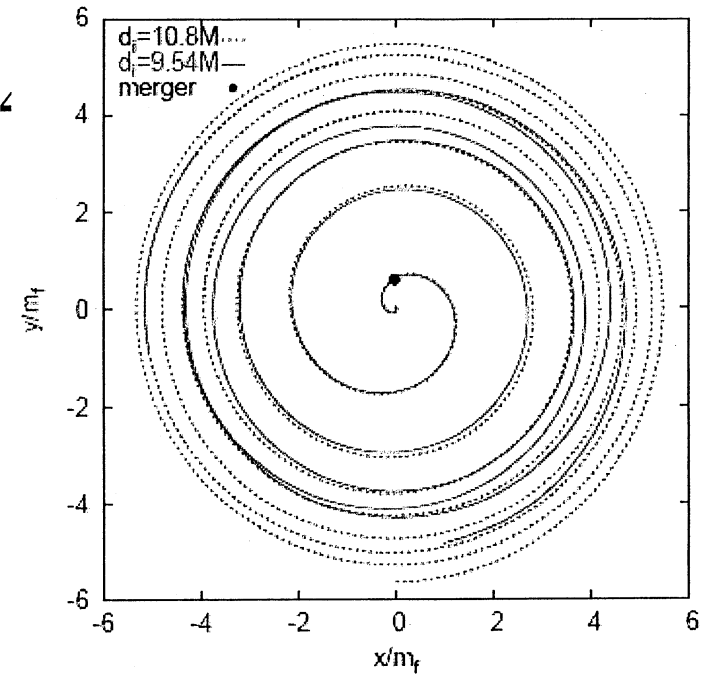
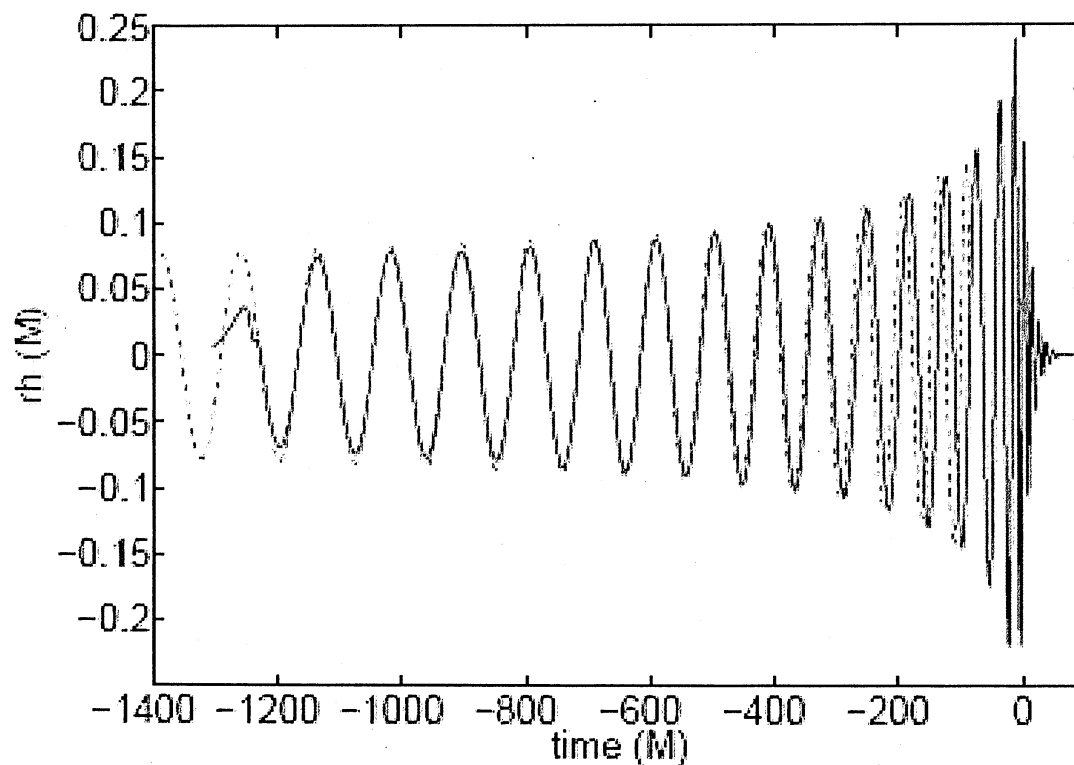
$$\text{Im}[\psi_4] \sim d^2/dt^2 h_x$$

*(Visualizations by Chris Henze, NASA/Ames)*

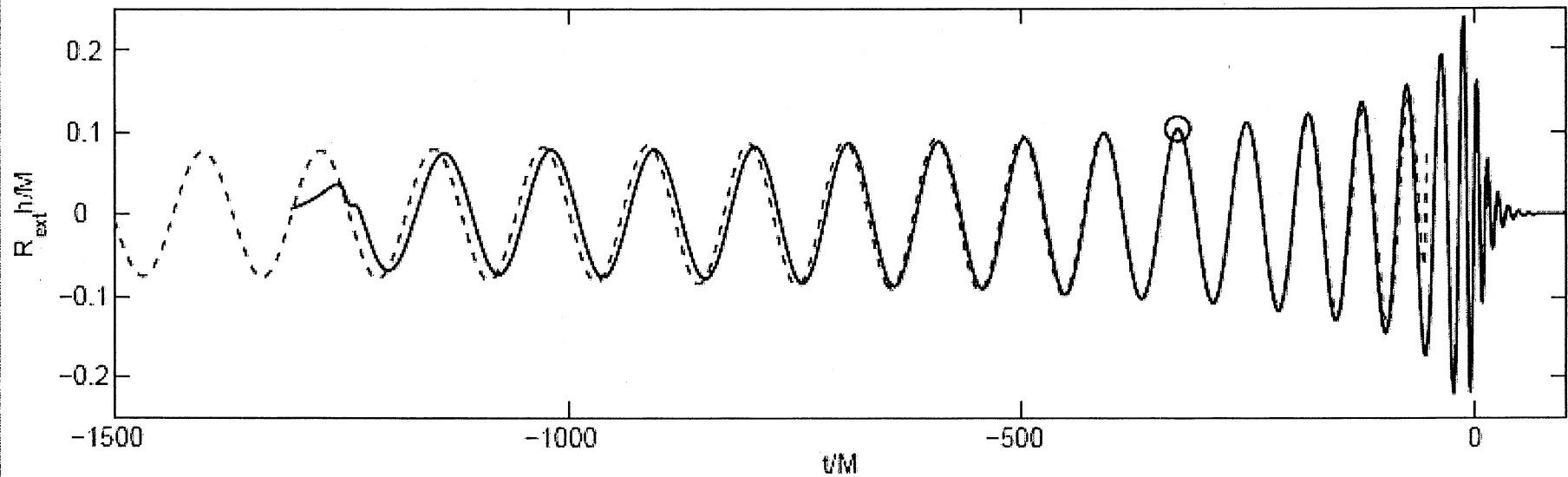


## *Longer runs, starting in late inspiral...*

- Baker, et al., gr-qc/0612117; gr-qc/0612024
- Evolve  $\sim 1200M$  and  $\sim 7$  orbits before merging
- Lower initial eccentricity  $e \sim 0.008$
- Validation of 3.5 PN in late inspiral



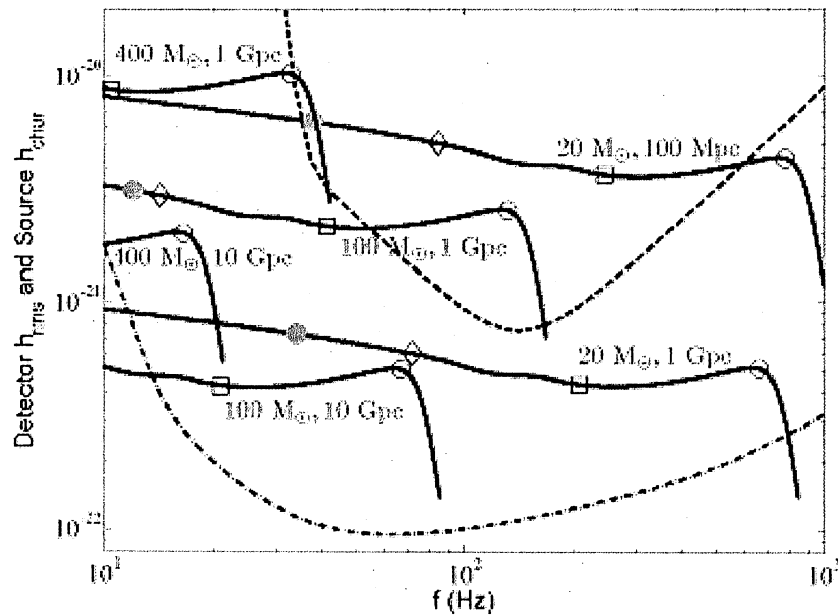
## *“Observing” the mergers...*



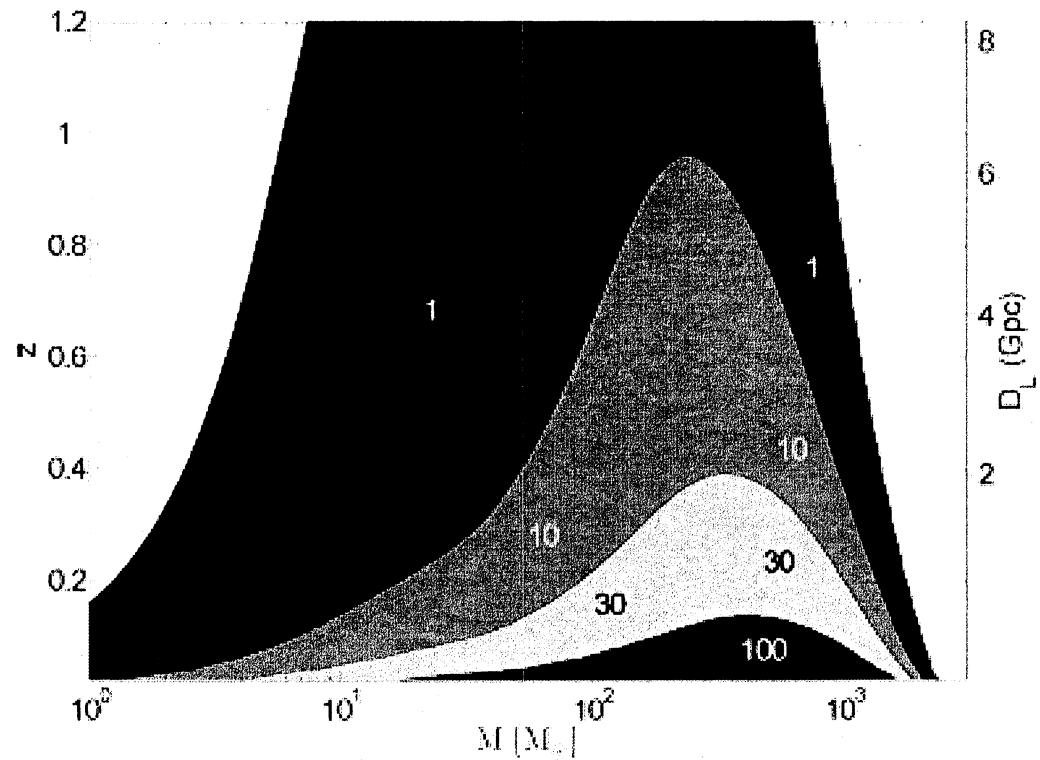
- **Baker, et al., gr-qc/0612117**
- **Make composite waveform**
- **Compare sensitivity, SNR for current and future detectors**

# Observing BH mergers w/ LIGO...

- Note these results are for equal mass, non-spinning BHs
- Unequal masses, spins will alter these results...



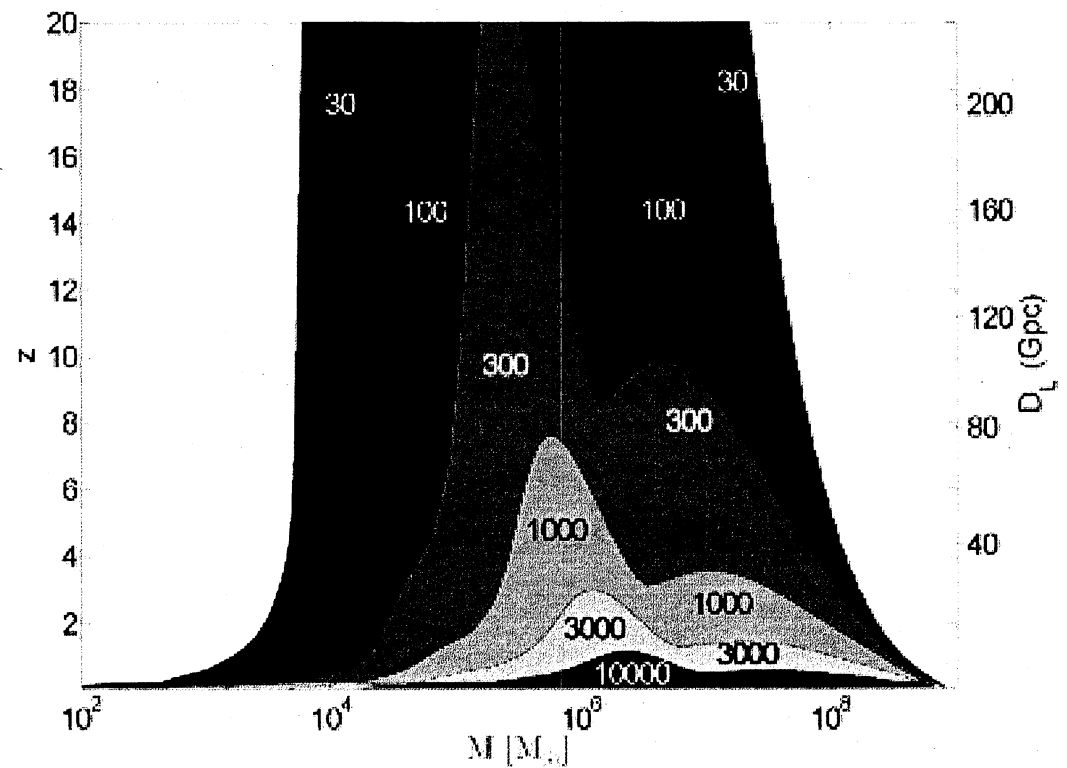
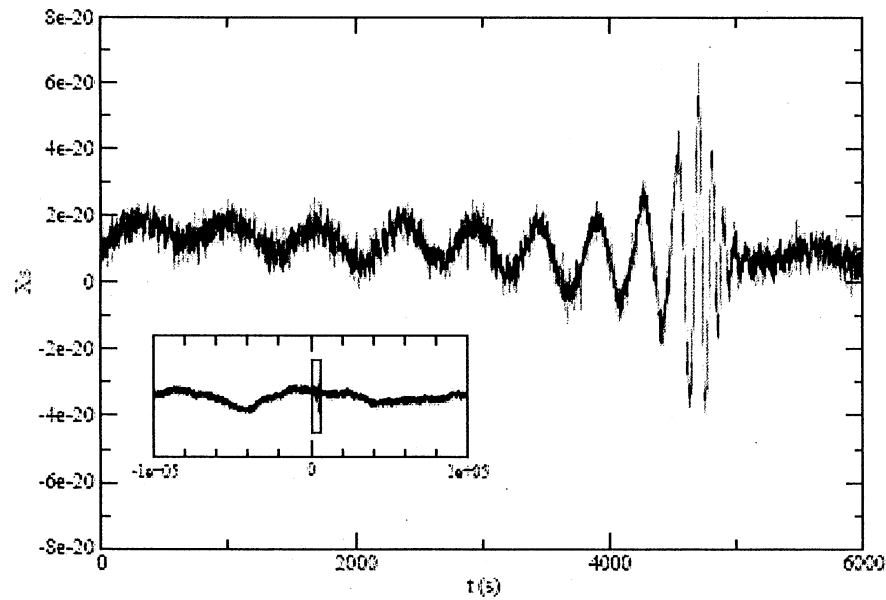
Advanced LIGO





# Observing MBH mergers with LISA....

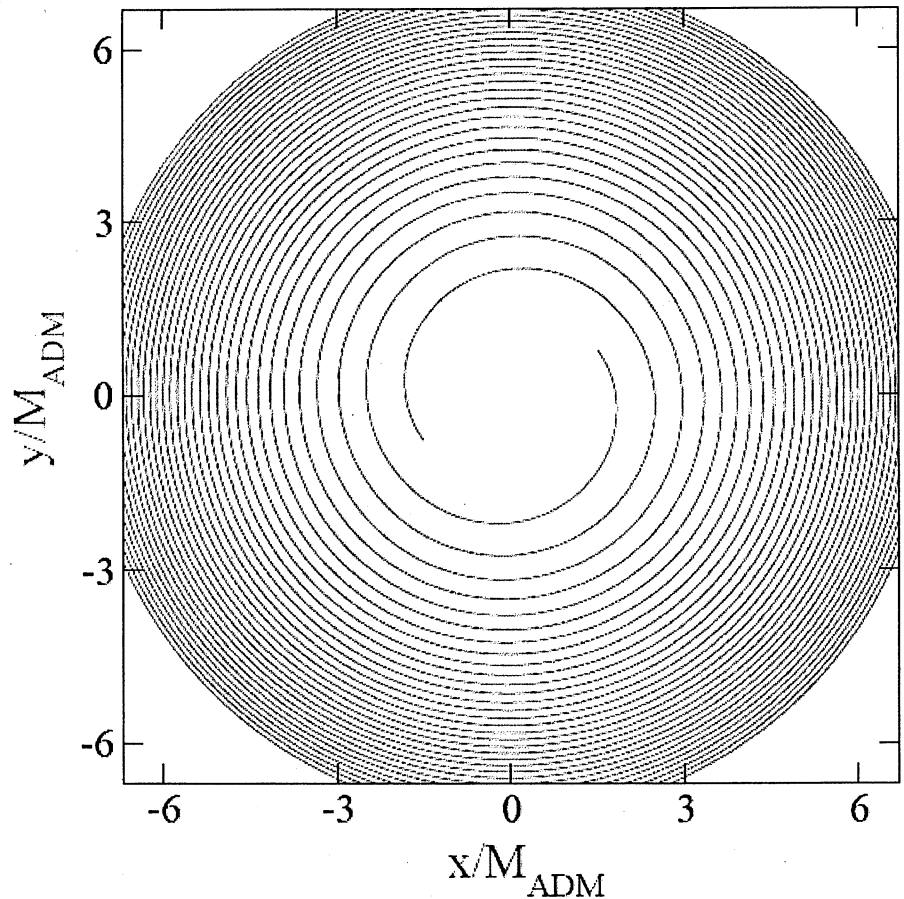
*Baker, et al., gr-qc/0612117*



## *New results for inspiral regime...*

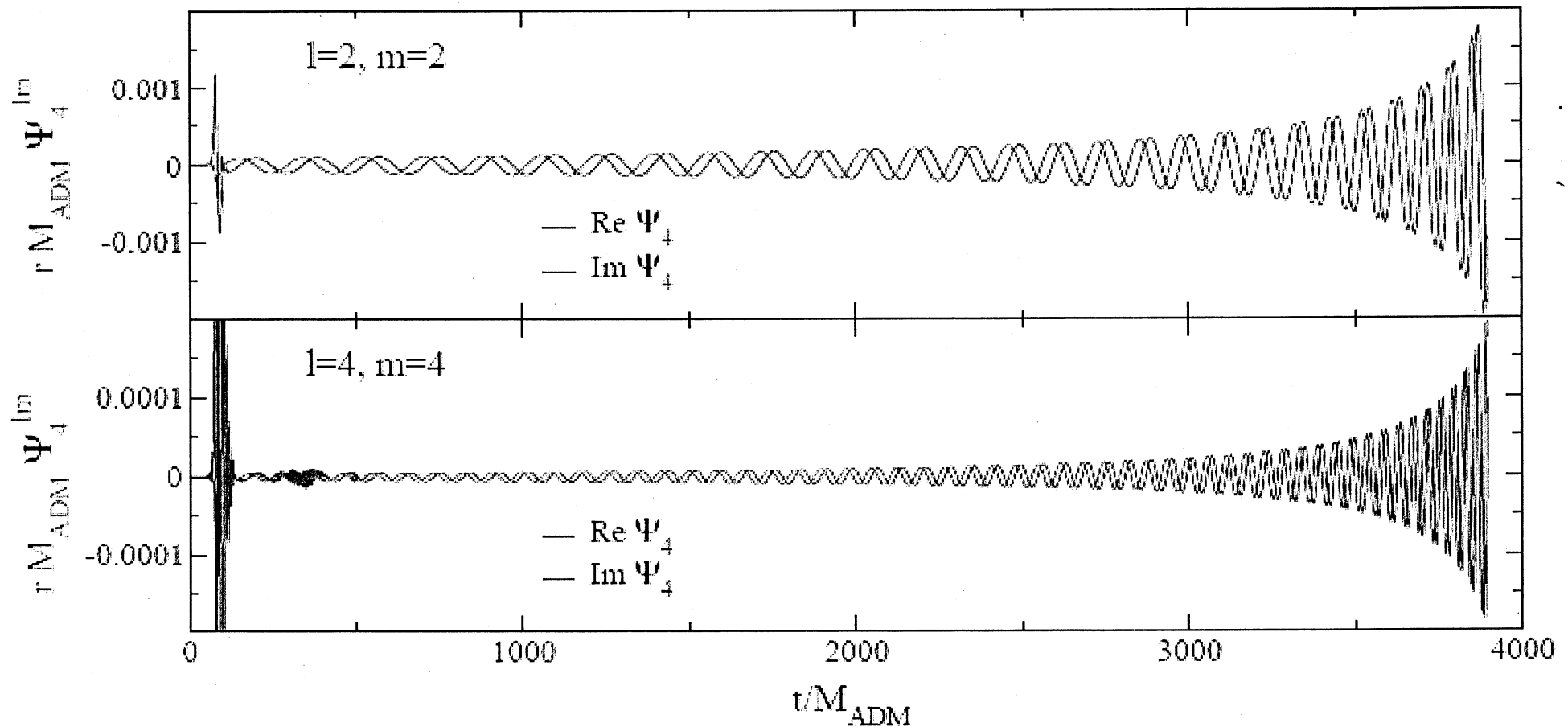
**Caltech/Cornell collaboration**

- **Use 1<sup>st</sup> order form of generalized harmonic formalism**
- **Multi-domain spectral code – very rapid convergence**
- **BHs are excised**
- **Rotating coordinates**
- **Evolve  $\sim 15$  orbits of inspiral**
- **Need to re-grid to handle merger and ringdown – work in progress**



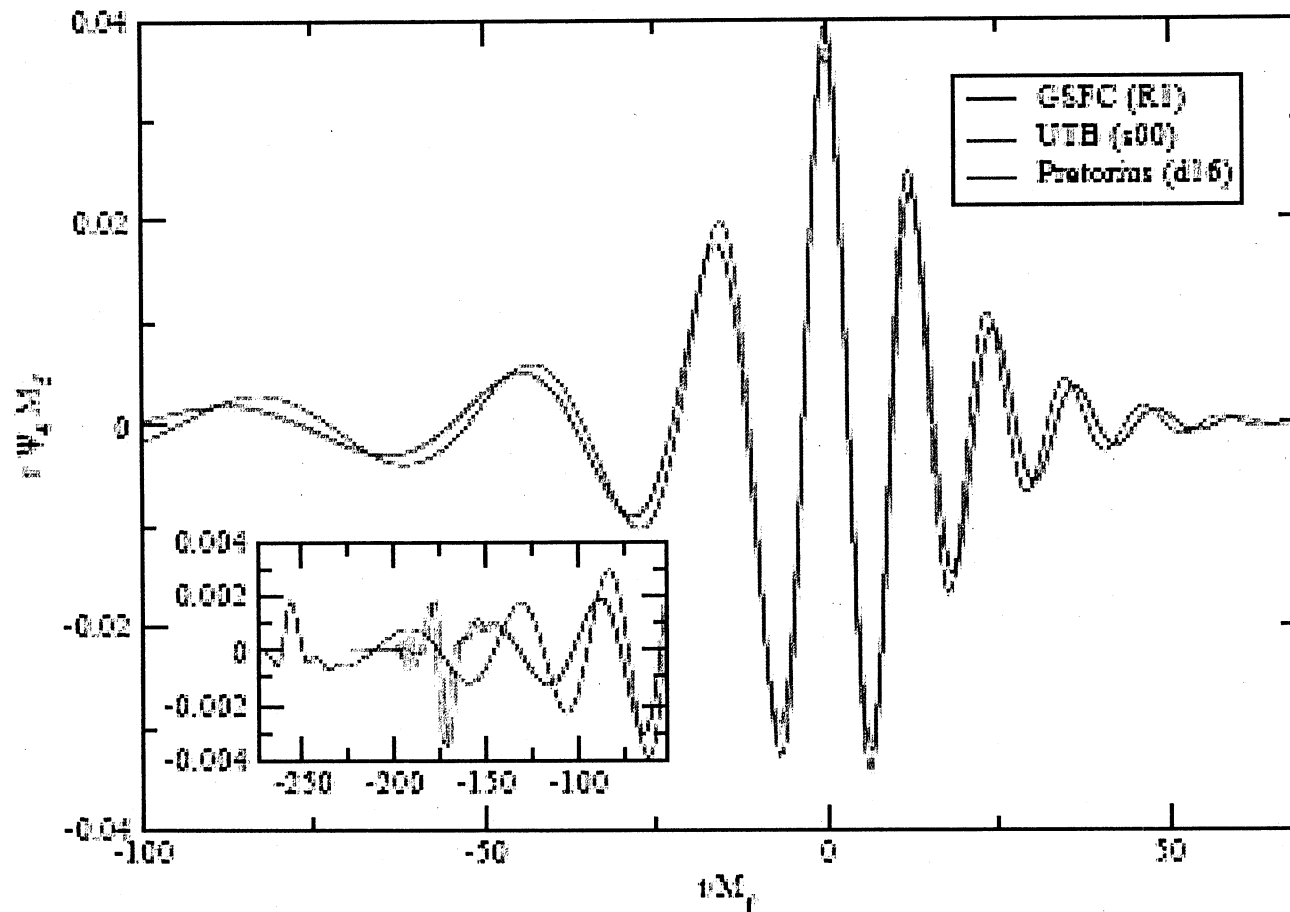
## *Long wavetrains...*

- Evolve for nearly 4000M
- Very low phase errors...< 0.1 radians over 15 orbits
- confirm results for accuracy of PN in during inspiral



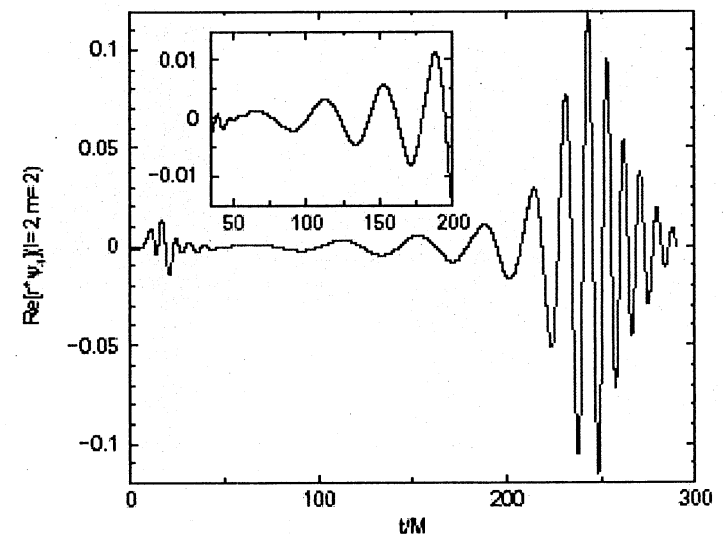
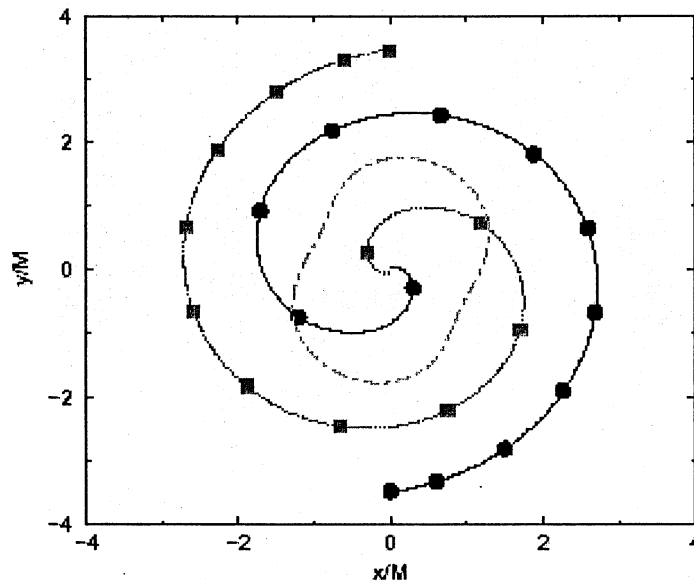
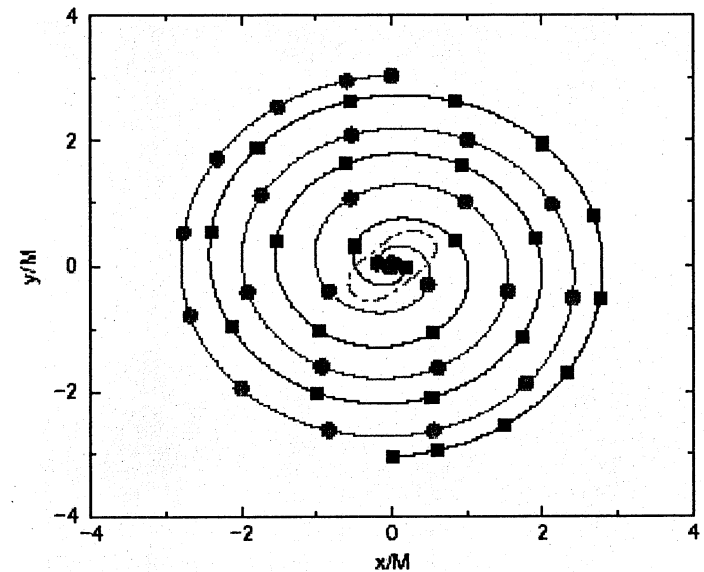
# Comparison of gravitational waveforms...

- Baker, Campanelli, Pretorius, Zlochower, gr-qc/0701016
- Compare GWs from equal mass, nonspinning case
- 3 different, independently-written codes



# *Evolutions of equal mass BHs with spin...*

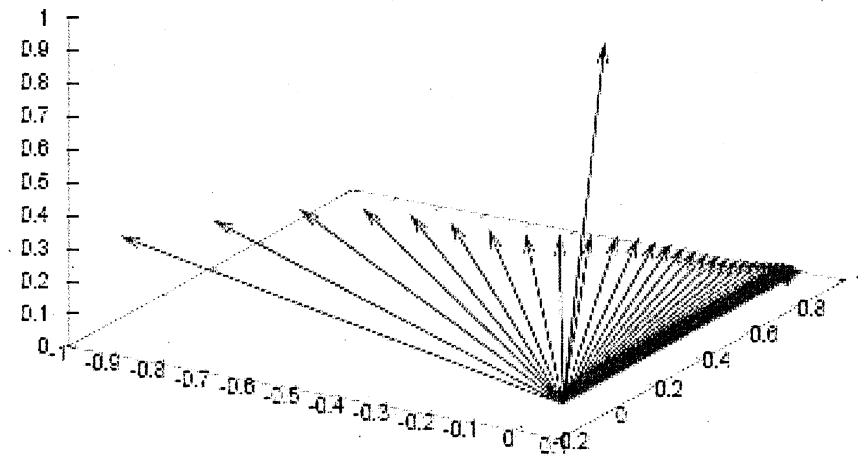
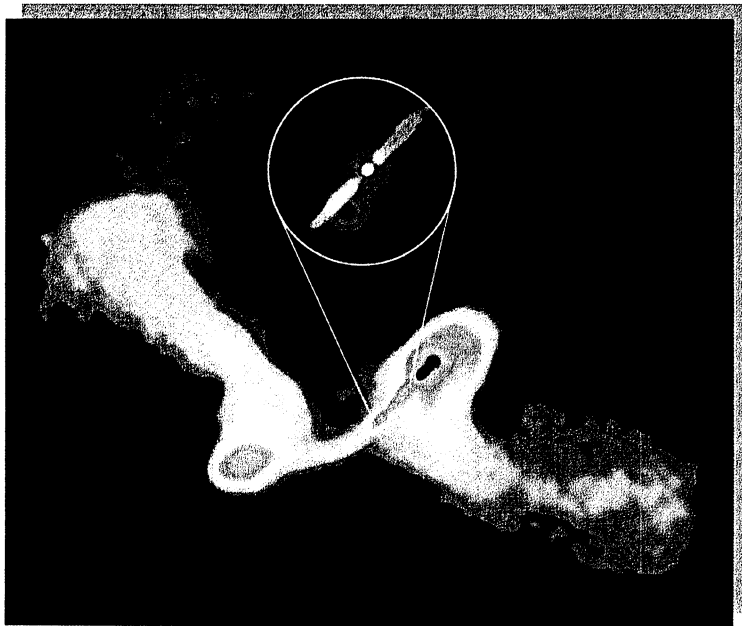
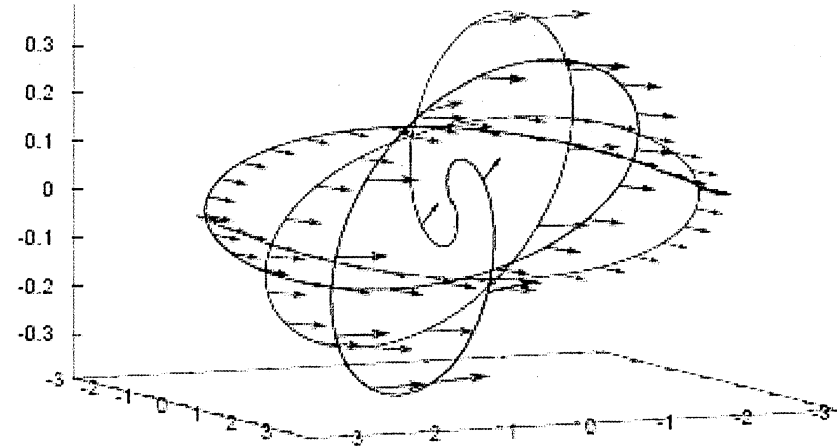
- Campanelli, et al., Phys.Rev. D74 (2006) 041501 (gr-qc/0604012)
- Moving punctures; 1<sup>st</sup> BBHs with spin
- Equal masses, each with  $a = 0.75 M$
- Initially  $M\Omega = 0.05 \rightarrow T_{\text{orbital}} \sim 125M$
- Anti/aligned  $\rightarrow$  attractive/repulsive
- Final  $a=0.9M$  (aligned),  $a=0.44M$  (anti)



# *A spin flip...*

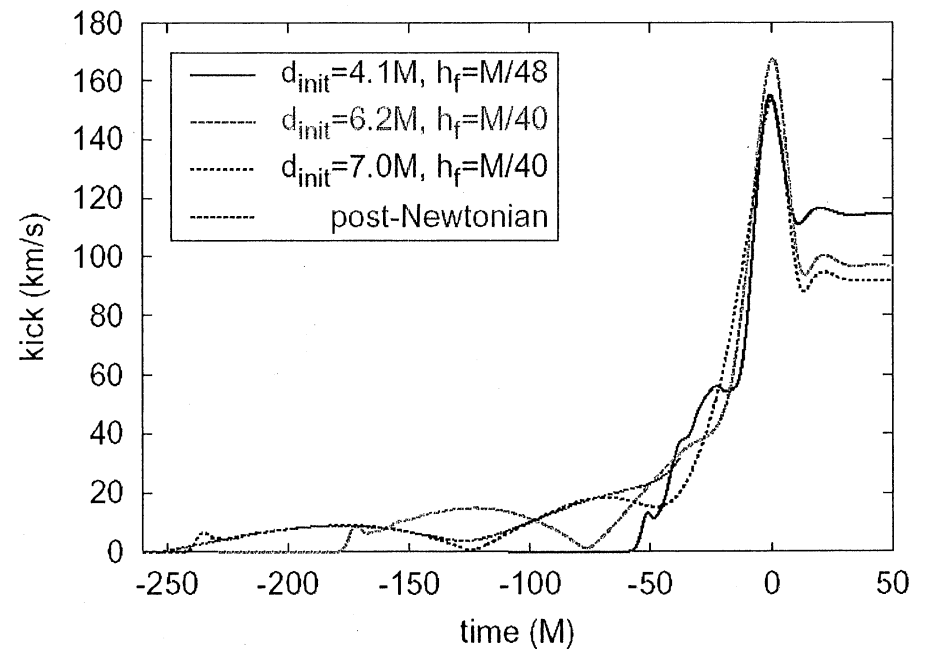
Campanelli, et al., gr-qc/0612076

- Equal masses and spins
- Parallel spins in orbital plane
- Spins precess by  $> 90^\circ$   $\rightarrow$
- Final spin of remnant “flips” by  $\sim 72^\circ$  from initial spins
- Also evolve w/ parallel spins at  $45^\circ$  to orbital plane
- Remnant BH has spin flipped by  $\sim 34^\circ$  from initial spins



## *Unequal mass BBH mergers...*

- When  $m_1 \neq m_2$ , the GW emission is asymmetric
- GWs carry momentum, so merged remnant BH suffers recoil ‘kick’
- Most of recoil occurs in strong gravity  $\rightarrow$  numerical relativity
- Baker, et al., ApJL, 653, L93 (2006) astro-ph/0603204:
  - $q = m_1/m_2 = 0.67$
  - widest separation run completes  $\sim 2.5$  orbits before merger
  - agrees w/ PN over most of 1<sup>st</sup> orbit to better than 1%
  - Overall, report kick values in the range  $v_{\text{kick}} = (86 - 97)\text{km/s}$
- Gonzales, et al, gr-qc/0610154
  - Ran series of runs, w/ mass ratios in the range  $0.253 \leq q \leq 1$
  - Find max kick  $V_{\text{max}} = 175.7 \pm 11 \text{ km/s}$  for  $q = 0.36 \pm 0.03$



# *Recoiling from mergers of spinning BHs...*

**Astrophysical BHs are spinning...how will this impact the kicks?**

**Many new results...**

- Herrmann, et al., gr-qc/0701143
  - $q = 1$ , spins anti/aligned with orbital angular momentum
  - $a/m = 0.2, 0.4, 0.6, 0.8 \rightarrow v_{\text{kick}}$  up to  $\sim 400$  km/s
- Koppitz, et al., gr-qc/0701163
  - $q \sim 1$ ,  $a/m \sim 0.15$ , spins anti/aligned  $\rightarrow v_{\text{kick}}$  up to  $\sim 250$  km/s
- Campanelli, et al., gr-qc/0701164
  - $q = 0.5$ , spinning larger BH  $a/m = 0.885$  with spin at  $-45$  deg to orbital plane, orbits nonspinning smaller BH  $\rightarrow v_{\text{kick}} \sim 454$  km/s
- Gonzalez, et al., gr-qc/0702052
  - $q = 1$ ,  $a/m \sim 0.8$ , spins in orbital plane, oppositely directed, chosen to maximize kick  $\rightarrow$  get  $v_{\text{kick}} \sim 2500$  km/s !!
- Baker, et al., astro-ph/0702390
  - model  $v_{\text{kick}}$  for spins aligned/anti to within  $\sim 10\%$

*$\rightarrow$  interesting parameter space...more studies to come*





## *Current status of BBH merger simulations...*

- Impressive recent progress on a broad front: many research groups, different codes, methods...
- Equal mass, nonspinning BBHs: several groups are now evolving for several orbits, followed by the plunge, merger, and ringdown
- There is general agreement on the *simple waveform shape* and that
  - total GW energy emitted in last few cycles  
 $\Delta E \sim (0.035 - 0.04)M$  (*depends on the number of orbits*)
  - final BH has spin  $a \sim 0.7M$
- Long runs now possible...~ 7 orbits before merger
- Applications to GW data analysis are beginning
- *Explosion* of work on nonequal mass and spinning BH mergers and the resulting kicks
  - Interesting parameter space
  - Important astrophysical applications...



# *The emerging picture....*

